## GCE AS/A level

WJEC CBAC

0984/01

## MATHEMATICS - S2 <br> Statistics

A.M. MONDAY, 10 June 2013
$1 ½$ hours

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)


## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer all questions.
Sufficient working must be shown to demonstrate the mathematical method employed.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. The random variable $X$ is normally distributed with mean 10 and standard deviation 2 .
(a) (i) Evaluate $P(X \leqslant 10 \cdot 5)$.
(ii) Given that $P(X \geqslant x)=0 \cdot 1$, find the value of $x$.
(b) The independent random variable $Y$ is normally distributed with mean 12 and standard deviation 3.
(i) Evaluate $P(X+2 Y<36)$.
(ii) Given that $X_{1}, X_{2}, X_{3}$ is a random sample from the distribution of $X$ and $Y_{1}, Y_{2}$ is a random sample from the distribution of $Y$, evaluate

$$
\begin{equation*}
P\left(X_{1}+X_{2}+X_{3}<Y_{1}+Y_{2}\right) . \tag{11}
\end{equation*}
$$

2. The amount of coffee dispensed into a cup by an automatic machine may be assumed to be normally distributed with mean $\mu \mathrm{ml}$ and standard deviation 4 ml . Successive amounts are independent. One morning, the machine was used 50 times and the amounts dispensed, $x \mathrm{ml}$, were measured. It was found that $\sum x=9980$.
(a) Find $95 \%$ confidence limits for $\mu$, giving your answers correct to one decimal place. [5]
(b) Determine the minimum sample size which would give a $95 \%$ confidence interval for $\mu$ having width less than 1.
3. A teacher in a large college wishes to investigate whether or not boys and girls perform equally well in examinations in practical mathematics. She therefore selects a random sample of 8 boys and 8 girls and gives them an examination.
The marks obtained were as follows.

$$
\begin{array}{ll}
\text { Boys } & 52,47,62,75,51,69,56,70 \\
\text { Girls } & 48,39,56,69,71,45,43,59
\end{array}
$$

You may assume that these are random samples from normal populations with a common standard deviation of $7 \cdot 5$.
(a) State suitable hypotheses for this investigation.
(b) Determine the $p$-value of these results and state your conclusion in context.
4. A politician believes that $40 \%$ of the voters support him but his agent believes that his support is greater than this. They decide to question a random sample of voters to try to resolve the situation.
(a) State suitable hypotheses.
(b) They question a random sample of 50 voters, of whom 25 state that they support the politician. Determine the $p$-value and state your conclusion in context at the $5 \%$ significance level.
(c) They now decide to question a random sample of 400 voters, of whom 181 state that they support the politician. Determine an approximate $p$-value and state your conclusion in context.
5. The number of accidents occurring per day along a certain stretch of road can be modelled by a Poisson distribution. The value of the mean $\mu$ has been $1 \cdot 2$ in the past but the local council has recently introduced a lower speed limit in the hope of reducing the value of $\mu$.
(a) State suitable hypotheses for testing whether or not lowering the speed limit has had the desired effect.
(b) It is decided to count the number of accidents, $x$, occurring in a 60-day period and to define the critical region as $x \leqslant 58$.
(i) Determine the significance level.
(ii) Given that the value of $\mu$ has actually fallen to $0 \cdot 8$, determine the probability of concluding that there has been no reduction in the value of $\mu$.
6. The radius $R$ of a circle is a continuous random variable that is uniformly distributed on the interval $[6,8]$.
(a) Let $C$ denote the circumference of the circle. Determine
(i) the mean and the variance of $C$,
(ii) $P(C \leqslant 45)$.
(b) Let $A$ denote the area of the circle. Determine
(i) $P(A \geqslant 150)$,
(ii) $E(A)$.

